Tosun Terzioğlu's Contributions to Mathematics

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Tosun Terzioğlu's field of study was *Functional Analysis*. His contributions concern basic results that affect the entire field of locally convex topological vector spaces and especially the structure theory of Fréchet spaces.

Considered to be one of the basic results of the field, the characterization of compact linear operators between two Banach spaces [1971] is crucial in terms of both content and applicability.

In his doctoral thesis, Terzioğlu elaborated on and developed the concept of *diametral dimension* in the context of locally convex spaces and explored the implementations of this concept. For instance, he showed that Schwartz spaces can be defined by means of this invariant. In this context, he described the Köthe space families which he named G_1 and G_∞, and showed that spaces are in a natural duality [1973 - 1976].

Kolmogorov diameters and various versions of diametral dimension occupy a significant place in Terzioğlu's lifelong studies. In the following years, he defined and developed *m-rectangular* invariants, an invariant stronger than diametral dimension, in collaboration with P. A. Chalov and V. P. Zahariuta. They used these invariants effectively in distinguishing some locally convex tensors / direct products [1995 - 2004].

In the second half of 1970s, Terzioğlu focused his studies on nuclear spaces introduced first by A. Grothendieck in 1955, expanded the concept of nuclearity with M. S. Ramanujan, and described the $\Lambda_{\mathbb{N}}(\alpha)$-nuclearity feature, and proved that for stable sequences (where $\alpha = (\alpha_n)$ for $\sup_n \alpha_{2n}/\alpha_n < \infty$ olan), Fréchet spaces having this feature overlap with the closed sub-spaces of $\Lambda_\infty(\alpha)^{\mathbb{N}}$ [1975].

Having made significant contributions to the structural theory of nuclear Köthe spaces with M. Alpseymen Kocatepe and other students of his, Terzioğlu remained one of the prominent contributors of this area following the paradigmatic shift which occurred in the 1980s. In 1985, Terzioğlu realized a technical yet very interesting study on the impact of the structural invariants developed by D. Vogt and his school on Kolmogorov diameters, and thus on diametral dimension. Setting out from this study, A. Aytuna, J. Krone and T. Terzioğlu came up with the notion of associated exponent sequences for Fréchet spaces which meet the *DN* and Ω features elaborated by the Vogt school [1989 - 1990]. In these joint studies, it was proved that nuclear Fréchet spaces whose diametral dimension overlaps with a stable nuclear infinite type power series space have a complemented subspace isomorphic to that power series space. On the other hand, in recent years, Terzioğlu [2013] started to study Fréchet spaces whose diametral dimension overlaps with stable nuclear finite-type power series spaces, and developed joint projects with P. Djakov and A. Aytuna in this field. Some of the results reached during these studies are soon to be published.

In the structural theory of nuclear Fréchet spaces, a dichotomy which may be traced back to A. Aytuna - T. Terzioğlu [1981] but was first formulated in Terzioğlu's joint work with Z. Nurlu [1989] opened up a vast field of research. This dichotomy can be summarized as "Either all linear continuous operators between two nuclear Fréchet spaces are compact (in other words, the two are radically different from one another), or these two spaces have common (and even complemented) sub-spaces and quotient spaces." The question of under which conditions this dichotomy holds, and the exploration of all aspects of this phenomenon were the focus of Terzioğlu's joint studies with S. Önal, V. P. Zahariuta, P. Djakov and M. Yurdakul which would last decades [1984 - 2004].

Starting from 1994, Terzioğlu conducted various joint studies with mathematicians such as V. P. Zahariuta, P. Djakov, A. Goncharov and P. A. Chalov on the classification of the complemented closed sub-spaces of nuclear direct and / or tensor product under which the factors of the product spaces become isomorphic. These studies which lasted until 2011 yielded important results in the context of tensor products of infinite type power series and duals.

In the 1970s, A. Pelczynski had posed the question whether the closed sub-spaces of those nuclear Fréchet spaces which have a base would also automatically have a base. This question remains open even for infinite type power series which occupy an important place in analysis.

A. Aytuna, J. Krone and T. Terzioğlu proved in 1998 that complementable closed stable sub-spaces of infinite type nuclear power series have a base.

Even today, this conclusion remains a comprehensive and useful instrument in finding the bases of certain Fréchet spaces, which appear naturally in the analysis.

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